Intro to Limits

1.1 EU 1.1 The concept of a limit can be used to understand the behavior of functions.
Students will be able to:

❤ Express limits symbolically using correct notation.
❤ Interpret limits expressed symbolically.
❤ Estimate limits of functions.
Limits

We write a limit using this notation:

\[ \lim_{{x \to c}} f(x) = L \]

As you approach an x value from the right and the left, a limit finds the height that the function intends to reach.

The limit exists only if you approach the same height from the left and the right & does not approach without bound (infinity).
Numbers, Graphs, and Algebra

When finding limits, you can find the answer using three techniques:

- Numerically (table)
- Graphically (graphing)
- Algebraically
Limits Approaching...

♥ The limit of a function refers to the value that the function approaches, not the actual value (if any).

♥ Look at what the graph is approaching, not the closed circle.
Numerically & Graphically

\[ \lim_{x \to c} f(x) = L \]

♥ Numerically:
♥ Make a table of x values that are really close to c from the left and the right.

♥ Find the limit numerically, then graphically.

1. \( \lim_{x \to 4} x^2 \)

\[ \begin{array}{c|c|c}
    x & f(x) & \text{Graph}
    \hline
    3 & 9 & \uparrow
    3.5 & 12.25 & \uparrow
    4 & 16 & \uparrow
    4.5 & 20.25 & \uparrow
    5 & 25 & \uparrow
    \end{array} \]

1. 16
Find the limit numerically & graphically

2. \( \lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} \)
Find the limit numerically & graphically

3. \( \lim_{{x \to 0}} \frac{\sin x}{x} \)
Graphically

1. \( \lim_{x \to -1} f(x) \)
2. \( \lim_{x \to 0} f(x) \)
3. \( f(0) \)
4. \( \lim_{x \to 2} f(x) \)

\[
\begin{array}{c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 & 2 \\
\hline
 f(x) & 3 & 2 & 1 & 2 & 3 \\
\end{array}
\]
Limits That Do Not Exist

DNE limits:
- do not approach the same height from the left and the right
- approach $\infty$ or $-\infty$
Find the limits numerically and graphically

1. \( \lim_{x \to 0} \frac{|x|}{x} \)

2. \( \lim_{x \to 0} \frac{1}{x^2} \)

1. dne
2. dne
Find the limit graphically

1. \( \lim_{{x \to 1}} f(x) \)
2. \( \lim_{{x \to 4}} f(x) \)
3. \( f(1) \)
4. \( f(4) \)
5. \( \lim_{{x \to 2}} f(x) \)

1. 1  
2. 2  
3. 2  
4. dne  
5. dne
One-Sided Limits

♥ For a limit to exist, the function must approach the same value (height) from both sides.

♥ One-sided limits approach from either the left or right side only.
One-Sided Limits

Right hand limit:
\[ \lim_{x \to c^+} f(x) \]
the limit of \( f \) as \( x \) approaches \( c \) from the right

Left hand limit:
\[ \lim_{x \to c^-} f(x) \]
the limit of \( f \) as \( x \) approaches \( c \) from the left
Find the limits

1. \( \lim_{{x \to 1^+}} f(x) \)
2. \( \lim_{{x \to 1^-}} f(x) \)
3. \( \lim_{{x \to 1}} f(x) \)
4. \( f(1) \)

1.1 \quad 2.0
3. dne \quad 4.1
Find the limits

5. \( \lim_{{x \to 2^+}} f(x) \) 
6. \( \lim_{{x \to 2^-}} f(x) \)

7. \( \lim_{{x \to 2}} f(x) \) 
8. \( f(2) \)

\[ \begin{array}{cc}
5.1 & 6.1 \\
7.1 & 8.2 \\
\end{array} \]
Find the limits

9. \( \lim_{x \to 3^+} f(x) \)
10. \( \lim_{x \to 3^-} f(x) \)
11. \( \lim_{x \to 3} f(x) \)
12. \( f(3) \)

\[
\lim_{x \to 3} f(x) \to 2 \\
\lim_{x \to 3^-} f(x) \to 2 \\
\lim_{x \to 3^+} f(x) \to 2 \\
f(3) = 2
\]
Find the limits

13. \( \lim_{{x \to 4^+}} f(x) \)
14. \( \lim_{{x \to 4^-}} f(x) \)
15. \( \lim_{{x \to 4}} f(x) \)
16. \( f(4) \)

13. dne 14. 1
15. dne 16. 1